Electrostatics of Conductors

Objectives

After going through this lesson, the learners will be able to

- Understand the electrostatics of conductors
- Distinguish between conductors and insulators
- Conceptualize electrostatic shielding
- Define the nature of dielectrics.
- Distinguish between the polar and non- polar molecules
- Explain the phenomenon of electric polarisation

Content Outline

- Unit syllabus
- Module wise distribution of unit syllabus
- Words you must know
- Introduction
- Conductors and Insulators
- Free charges and bound charges inside a conductor
- Electrostatics of conductors
- Dielectrics and Electric Polarization
- Summary

Unit Syllabus

Unit 1: Electrostatics:

Chapter-1: Electric Charges and Fields

Electric Charges; Conservation of charge, Coulomb's law-force between two point charges, forces between multiple charges; superposition principle and continuous charge distribution.

Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field.

Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

Chapter-2: Electrostatic Potential and Capacitance

Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges; equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field.

Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics and electric polarization, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor.

Module Wise Distribution of Unit Syllabus - 11 Modules

The above unit is divided into 11 modules for better understanding.

Module 1	• Electric charge • Properties of charge • Coulombs' law • Characteristics of coulomb force • Constant and the intervening medium • Numerical
Module 2	• Forces between multiple charges • Superposition • Continuous distribution of charges numerical
Module 3	• Electric field E • Importance of field and ways of describing field • Point charges superposition of electric field • Numerical
Module 4	• Electric dipole • Electric field of a dipole • Charges in external field • Dipole in external field Uniform and non-uniform
Module 5	• Electric flux, • Flux density • Gauss theorem

	• Application of gauss theorem to find electric field • For a distribution of charges • Numerical
Module 6	• Application of gauss theorem Field due to field infinitely long straight wire • Uniformly charged infinite plane • Uniformly charged thin spherical shell (field inside and outside) • Graphs
Module 7	• Electric potential, • Potential difference, • Electric potential due to a point charge, a dipole and system of charges; • Equipotential surfaces, • Electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field. • Numerical
Module 8	• Conductors and insulators, • Free charges and bound charges inside a conductor. • Dielectrics and electric polarization
Module 9	• Capacitors and capacitance, • Combination of capacitors in series and in parallel • Redistribution of charges, common potential • Numerical
Module 10	• Capacitance of a parallel plate capacitor with and without dielectric medium between the plates • Energy stored in a capacitor
Module 11	• Typical problems on capacitors

Module 8

Words You Must Know

Let us remember the words we have been using in our study of this physics course.

- **Electric Charge:** Electric charge is an intrinsic characteristic of many of the fundamental particles of matter that gives rise to all electric and magnetic forces and interactions.
- **Conductors:** Some substances readily allow passage of electricity through them, others do not. Those which allow electricity to pass through them easily are called conductors. They have electric charges (electrons) that are comparatively free to move inside the material. Metals, human and animal bodies and earth are all conductors of electricity.
- **Insulators:** Most of the non-metals, like glass, porcelain, plastic, nylon, wood, offer high opposition to the passage of electricity through them. They are called insulators.
- **Point Charge:** When the linear size of charged bodies is much smaller than the distance separating them, the size may be ignored and the charge bodies can then be treated as point charges.
- **Coulomb's Force:** It is the electrostatic force of interaction between the two point charges.
- **Linear charge density:** The linear charge density λ is defined as the charge per unit length.
- **Surface charge density:** The surface charge density σ is defined as the charge per unit surface area.
- **Volume charge density:** The volume charge density ρ is defined as the charge per unit volume.
- **Superposition Principle:** For an assembly of charges q_1, q_2, q_3, ..., the force on any charge, say q_1, is the vector sum of the force on q_1 due to q_2, the force on q_1 due to q_3, and so on. For each pair, the force is given by Coulomb's law for two point charges.
- **Electric Field:** A region around a charged particle or object within which a force would be experienced by a charged particle or object.
- **Source and test charge:** The charge, which is producing the electric field, is called a source charge and the charge, which tests the effect of a source charge, is called a test charge.
- **Uniform Field:** A uniform electric field is one whose magnitude and direction is same at all points in space and it will exert same force of a charge regardless of the position of charge.

- **Non uniform field:** we know that electric field of point charge depends upon location of the charge. Hence has different magnitude and direction at different points. We refer to this field as non-uniform electric field.
- **Principle of superposition of fields:** Electric field intensity E at any point P due to all n point charges will be equal to the vector sum of electric field intensities E_1 ,E_2, E_3……E_n produced by individual charges at the point P. Hence $E = E_1 + E_2 + \ldots + E_n$
- **Torque:** Torque is the tendency of a force to rotate an object about an axis. The turning effect of force is called torque.
- **Electric field lines:** An electric field line is a curve drawn in such a way that the tangent at each point on the curve gives the direction of the electric field at that point.
- **Surface charge density in terms of area element:** The surface charge density σ at the area element Δs is given by $\sigma = \frac{\Delta Q}{\Delta s}$
- **Area vector:** The area element vector ΔS at a point on a closed surface equals $\Delta S\ \hat{n}$ where ΔS is the magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward normal at that point.
- **Gauss's law:** The flux of the electric field through any closed surface S is $1/\varepsilon_0$ times the total charge enclosed by that surface.
- **Gaussian surface:** The closed surface that we need to choose for applying Gauss's law to a particular charge distribution is called the Gaussian surface.
- **Electrostatic potential,** the electric potential at any point in an electric field, is defined as the work done in bringing a unit positive test charge from infinity to that point without acceleration.
- **Potential difference** between two points in an electric field is defined as the work done in bringing unit positive charge from one point to the other.
- **Equipotential surface** is the surface having the same potential at each point, the surface of a conductor in equilibrium is an equipotential surface. The surface separation decreases as we go towards a stronger field. An equipotential surface is a surface over which potential has a constant value. For a point charge, concentric spheres centred at a location of the charge are equipotential surfaces. The electric field E at a point is perpendicular to the equipotential surface through the point. E is in the direction of the steepest decrease of potential.
- **Electric potential for a system of point charges** it is the sum of all potentials due to individual charges.

- **Potential energy stored in a system of charges** is the work done (by an external agency) in assembling the charges at their locations.
- **Electric potential energy of a dipole in uniform electric field = - p.E**

Introduction

As we already know from our earlier study of previous modules, a metal rod held in hand and rubbed with wool will not show any sign of being charged. However, if a metal rod with a wooden or plastic handle is rubbed without touching its metal part, it shows signs of charging.

Suppose we connect one end of a copper wire to a neutral pith ball and the other end to a negatively charged plastic rod. We will find that the pith ball acquires a negative charge. If a similar experiment is repeated with a nylon thread or a rubber band, no transfer of charge will take place from the plastic rod to the pith ball. Why does the transfer of charge not take place from the rod to the ball?

In this module we will go beyond the general definition of conductors and insulators.

Conductors and Insulators

Some substances readily allow passage of electricity through them, others do not. Those which allow electricity to pass through them easily are called **conductors.** They have electric charges (electrons) that are comparatively free to move inside the material. Metals, human and animal bodies and earth are conductors.

Most of the non-metals like glass, porcelain, plastic, nylon, wood offer high resistance to the passage of electricity through them. They are called **insulators.** Most substances fall into one of the two classes stated above. There is a third category called **semiconductors,** which offer high resistance to the movement of charges or behave like insulators, but sometimes and under certain conditions behave like conductors they are intermediate between the conductors and insulators. We will talk about semiconductors in detail in our Physics course 4.

In the broad category we consider materials as conductors and insulators.

The free electrons form a kind of ‘gas’; they collide with each other and with the + ions, and move randomly in different directions. Under the influence of an external electric field, they drift against the direction of the field. The positive ions made up of the nuclei and the bound electrons remain held in their fixed positions.

When some charge is transferred to a conductor, it readily gets distributed over the entire surface of the conductor.

In contrast, if some charge is put on an insulator, it stays at the same place.

This property of the materials tells you why a nylon or plastic comb gets electrified on combing dry hair or on rubbing, but a metal article like spoon does not.

The charges on metal flow through our body to the ground as both are conductors of electricity. When we bring a charged body in contact with the earth, all the excess charge on the body disappears by causing a momentary current to pass to the ground through the connecting conductor (such as our body).

This process of sharing the charges with the earth is called grounding or earthing.

Earthing provides a safety measure for electrical circuits and appliances. A thick metal plate is buried deep into the earth and thick wires are drawn from this plate; these are used in buildings for the purpose of earthing near the mains supply.

The electric wiring in our houses has three wires: live, neutral and earth. The first two carry movement of charges. Electric current from the power station and the third is earthed by connecting it to the buried metal plate. Metallic bodies of the electric appliances such as electric iron, refrigerator, TV are connected to the earth wire.

When any fault occurs or live wire touches the metallic body, the charge flows to the earth without damaging the appliance and without causing any injury to the humans; this would have otherwise been unavoidable since the human body is a conductor of electricity

In electrolytic conductors, the charge carriers are both positive and negative; but the situation in this case is more involved – the movement of the charge carriers is affected both by the external electric field as also by the so-called chemical forces. We shall restrict our discussion to metallic solid conductors.

Free Charges and Bound Charges inside a Conductor.

Conductors contain mobile charge carriers. In metallic conductors, these charge carriers are electrons. In a metal, the **outer (valence) electrons** part away from their atoms and are free to move. These electrons are free within the metal but not free to leave the metal. The free electrons form a kind of ‘gas’; they collide with each other and with the ions, and move randomly in different directions.

In an external electric field, they drift against the direction of the field. The positive ions made up of the nuclei and the bound electrons remain held in their fixed positions.

Electrostatics of conductors

We have been studying about charges and electrostatic fields; let us make a note of some important results regarding electrostatics of conductors.

Inside a conductor, electrostatic field is zero

Consider a conductor, neutral or charged. There may also be an external electrostatic field.

The electric field is zero everywhere inside the conductor. This fact can be taken as the defining property of a conductor. A conductor has free electrons.

As long as the external electric field is not zero, the free charge carriers or electrons would experience force and drift. In the static situation, the free charges would be distributed such that the electric field is zero everywhere inside.

Electrostatic field is zero inside a conductor.

- **At the surface of a charged conductor, electrostatic field must be normal to the surface at every point**

  If E were not normal to the surface, it would have some non-zero component along the surface. Free charges on the surface of the conductor would then experience force and move. In the static situation, therefore, E should have no tangential component.

  Thus the electrostatic field at the surface of a charged conductor must be normal to the surface at every point.

  (For a conductor without any surface charge density, the field is zero even at the surface.)

- **The interior of a conductor can have no excess charge in the static situation.**

  A neutral conductor has equal amounts of positive and negative charges in every small volume or surface element. When the conductor is charged, the excess charge can reside only on the surface in the static situation.

  This follows from Gauss's law. Consider any arbitrary volume element v inside a conductor. On the closed surface S bounding the volume element v, the electrostatic field is zero. Thus the total electric flux through S is zero.

Hence, by Gauss's law, there is no net charge enclosed by S. But the surface S can be made as small as you like, i.e., the volume v can be made vanishingly small.

This means there is no net charge at any point inside the conductor, and any excess charge must reside at the surface.

- **Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface**

  This follows from results 1 and 2 above. Since E = 0 inside the conductor and has no tangential component on the surface, no work is done in moving a small test charge within the conductor and on its surface. That is, there is no potential difference between any two points inside or on the surface of the conductor. Hence, the result shows if the conductor is charged, an electric field normal to the surface exists; this means potential will be different for the surface and a point just outside the surface.

  In a system of conductors of arbitrary size, shape and charge configuration, each conductor is characterized by a constant value of potential, but this constant may differ from one conductor to the other.

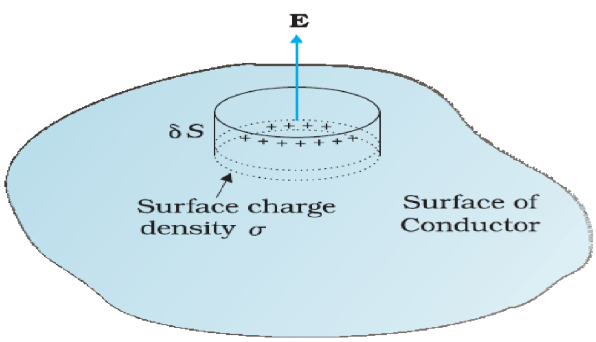
- **Electric field at the surface of a charged conductor**

$$E = \frac{\sigma \hat{n}}{\epsilon_0}$$

where σ is the surface charge density and

$\hat{n}$ is a unit vector normal to the surface in the outward direction.

To derive the result, choose a pill box (a short cylinder) as the Gaussian surface about any point P on the surface, as shown in Fig



The Gaussian surface (a pill box) chosen to derive an equation for electric field at the surface of a charged conductor.

The **pill box** is partly inside and partly outside the surface of the conductor. It has a small area of cross section δ S and negligible height.

Just inside the surface, the electrostatic field is zero; just outside, the field is normal to the surface with magnitude E.

Thus, the contribution to the total flux through the pill box comes only from the outside (circular) cross-section of the pill box. This equals ± E.δS (positive for σ > 0, negative for σ < 0), since over the small area δS, E may be considered constant and E and δS are parallel or antiparallel. The charge enclosed by the pill box is σ δS.

By Gauss's law:

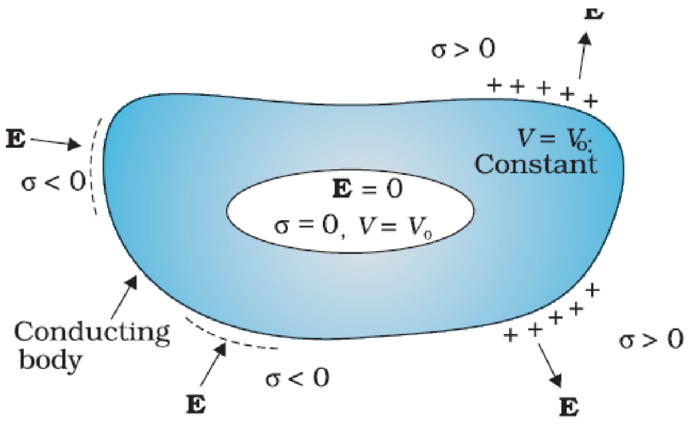
$$E\delta S = \frac{|\sigma|\delta S}{\varepsilon_0}$$

$$E = \frac{|\sigma|}{\varepsilon_0}$$

This equation is true for both signs of σ. For σ > 0, the electric field is normal to the surface outward; for σ < 0, electric field is normal to the surface inward.

- **Electrostatic shielding**

  Consider a conductor with a cavity, with no charges inside the cavity. A remarkable result is that the electric field inside the cavity is zero, whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external fields in which it might be placed. We have proved a simple case of this result already: the electric field inside a charged spherical shell is zero.



The proof of the result for the shell makes use of the spherical symmetry of the shell as discussed in module 6. But the vanishing of electric fields in the (charge-free) cavity of a conductor is, as mentioned above, a very general result. A related result is that even if the conductor is charged or charges are induced on a neutral conductor by an external field, all charges reside only on the outer surface of a conductor with cavity.

From the Figure note the important implication of the results.

Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence:

The field inside the cavity is always zero. This is known as electrostatic shielding.

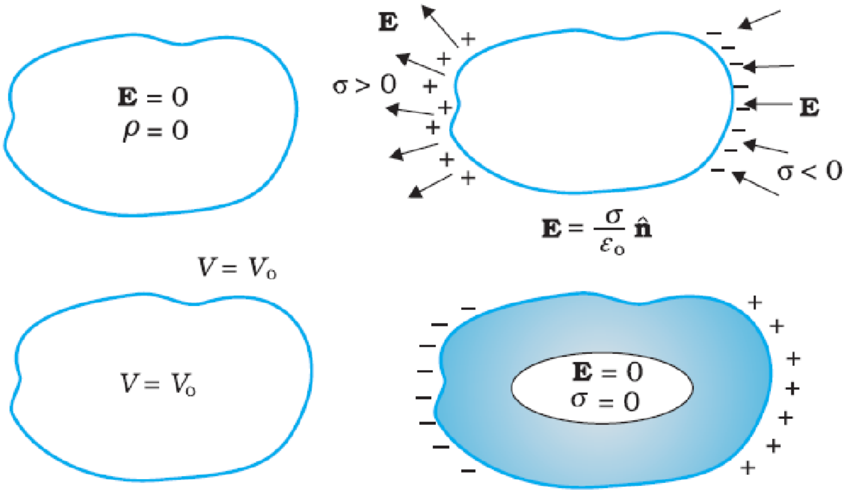
The effect can be made use of in protecting sensitive instruments from outside electrical influence.

Try This Out

Take a stainless steel container with a lid and big enough to hold a mobile phone call the mobile number now place the ringing phone inside the box, close the lid.

There is no sound as the connection is broken and the phone is shielded from outside EM waves

The figure below gives a summary of the important electrostatic properties of a conductor.



Example:

a. **A comb run through one's dry hair attracts small bits of paper. Why?**
b. **What happens if the hair is wet or if it is a rainy day? (Remember, a paper does not conduct electricity.)**
c. **Ordinary rubber is an insulator. But special rubber tires of aircraft are made slightly conducting. Why is this necessary?**
d. **Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why?**
e. **A bird perches on a bare high power line, and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?**

Solution:

a. This is because the comb gets charged by friction. The molecules in the paper get polarized by the charged comb, resulting in a force of attraction. If the hair is wet, or if it is a rainy day, friction between hair and the comb reduces. The comb does not get charged and thus it will not attract small bits of paper.

b. To enable them to conduct charge (produced by friction) to the ground; as too much of static electricity accumulated may result in spark and result in fire.

c. Reason similar to (b).

d. Current passes only when there is a difference in potential.

e. When a bird is perched on a bare high power line , the circuit does not complete between the bird and the earth therefore , nothing happens to the bird, but when a man standing on the ground touches the same line, the circuit between the man and earth goes completed.

Dielectrics and Polarisation

Dielectrics are non-conducting substances or another name for insulators. In contrast to conductors, they have no (or negligible number of) charge carriers.

When a conductor is placed in an external electric field then the free charge carriers move and charge distribution in the conductor adjusts itself in such a way that the electric field due to induced charges opposes the external field within the conductor. This happens until, in the static situation, the two fields cancel each other and then the electrostatic field in the conductor is zero.

In a dielectric, this free movement of charges is not possible.

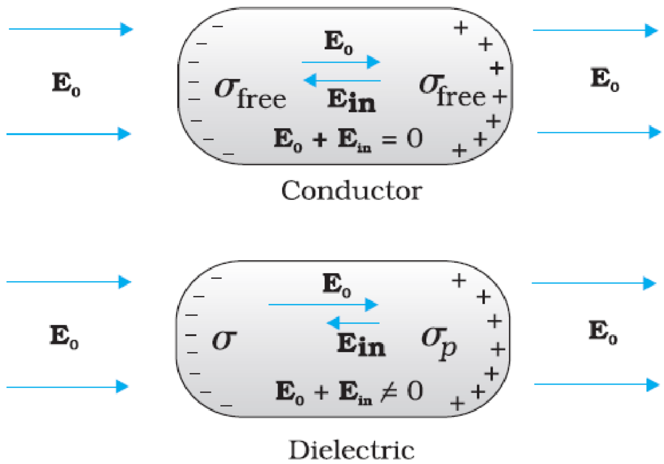


Fig shows difference in behavior of a conductor and an insulator in an external electrical field

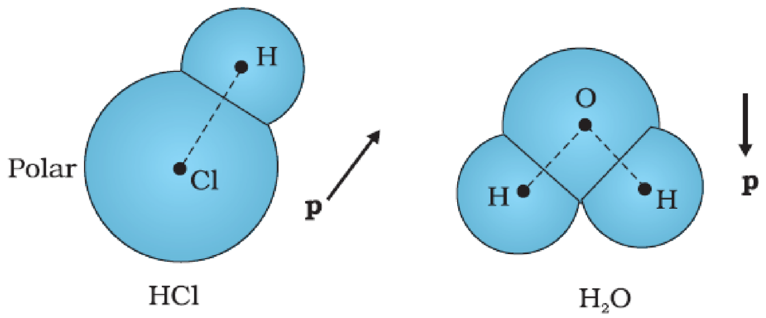
It turns out that the external field induces dipole moment by stretching or re-orienting molecules of the dielectric. The collective effect of all the molecular dipole moments is net charges on the surface of the dielectric which produce a field that opposes the external field. Unlike in a conductor, however, the opposing field induced does not exactly cancel the external field. It only reduces it.

The extent of the effect depends on the nature of the dielectric. To understand the effect, we need to look at the charge distribution of a dielectric at the molecular level.

Polar molecule

A polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field).

Such molecules have a permanent dipole moment. An ionic molecule such as HCl or a molecule of water (H_2O) are examples of polar molecules.

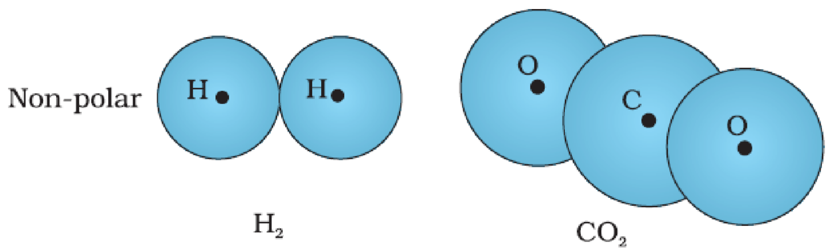


Non-polar molecules

A nonpolar molecule, the centers of positive and negative charges coincide.

The molecule then has no permanent (or intrinsic) dipole moment.

Examples of non-polar molecules are oxygen (O_2) and hydrogen (H_2) molecules which, because of their symmetry, have no dipole moment.



In an external electric field, the positive and negative charges of a non- polar molecule are displaced in opposite directions. The displacement stops when the external force on the constituent charges of the molecule is balanced by the restoring force (due to internal fields in the molecule).

Ionic polarization is polarization caused by relative displacements between positive and negative ions in ionic crystals (for example, NaCl)

The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarized by the external field. We consider only the simple situation when the induced dipole moment is in the direction of the field and is proportional to the field strength.

The induced dipole moments of different molecules add up giving a net dipole moment of the dielectric in the presence of the external field.

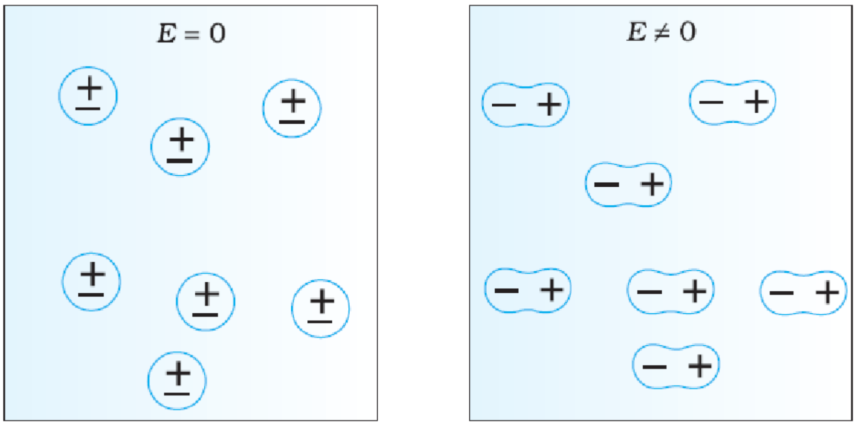
A dielectric with polar molecules also develops a net dipole moment in an external field, but for a different reason.

In the absence of any external field, the different permanent dipoles are oriented randomly due to thermal agitation; so the total dipole moment is zero.

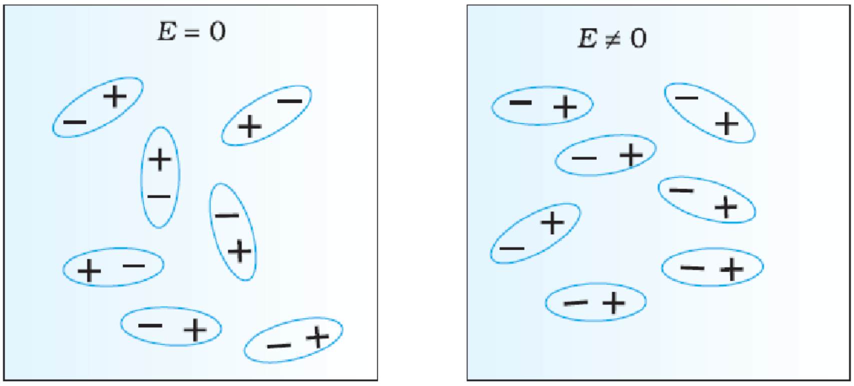
When an external field is applied, the individual dipole moments tend to align with the field.

When summed over all the molecules, there is then a net dipole moment in the direction of the external field, i.e., the dielectric is polarized.

The extent of polarization depends on the relative strength of two mutually opposite factors: the dipole potential energy in the external field tending to align the dipoles with the field and thermal energy tending to disrupt the alignment. There may be, in addition, the 'induced dipole moment' effect as for non-polar molecules, but generally the alignment effect is more important for polar molecules.



(a) Non-polar molecules



(b) Polar molecules

Thus in either case, whether polar or non-polar, a dielectric develops a net dipole moment in the presence of an external field. The dipole moment per unit volume is called **polarization** and is denoted by P.

For linear isotropic dielectrics

$P = \varepsilon_0 \chi_e E$

where χ_e is a constant characteristic of the dielectric and is known as the **electric susceptibility of the dielectric medium.**

Linear isotropic dielectrics means A dielectric medium is said to be linear when χ_e is independent of $\vec{E}$ and the medium is homogeneous if χ_e is also independent of space coordinates. A linear homogeneous and isotropic medium is called a simple medium and for such a medium the relative permittivity is a constant.

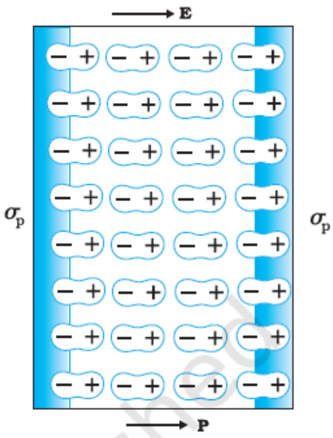
Dielectric constant ε_r may be a function of space coordinates. For anisotropic materials, the dielectric constant is different in different directions of the electric field

Now the question arises: how does the polarized dielectric modify the original external field inside it?

Let us consider, for simplicity, a rectangular dielectric slab placed in a uniform external field E0 parallel to two of its faces. The field causes a uniform polarization P of the dielectric. Thus every volume element Δv of the slab has a dipole moment P Δv in the direction of the field. The volume element Δv is macroscopically small but contains a very large number of molecular dipoles. Anywhere inside the dielectric, the volume element Δv has no net charge (though it has net dipole moment). This is, because, the positive charge of one dipole sits close to the negative charge of the adjacent dipole.

However, at the surfaces of the dielectric normal to the electric field, there is evidently a net charge density. As seen in Fig., the positive ends of the dipoles remain un-neutralized at the right surface and the negative ends at the left surface. The unbalanced charges are the induced charges due to the external field.

The electric susceptibility χ_e of a dielectric material is a measure of how easily it polarizes in response to an external electric field. This, in turn, determines the electric permittivity of the material.



A uniformly polarised dielectric amounts to induced surface charge density, but no volume charge density.

Thus the polarized dielectric is equivalent to two charged surfaces with induced surface charge densities, say **σ p and – σ p.** Clearly, the field produced by these surface charges opposes the external field. The total field in the dielectric is, thereby, reduced from the case when no dielectric is present. We should note that the surface charge density ± σ p arises from bound (not free charges) in the dielectric.

In electricity, the electric susceptibility is a dimensionless proportionality constant that indicates the degree of polarization of a dielectric material in response to an applied electric field,

It can be defined as the ratio of polarization to electric field strength in the dielectric

The units of electric susceptibility may be derived as

$$\frac{C}{m^2} \div \frac{N}{C} = \frac{C^2}{Nm^2}$$

Do you think temperature may change the susceptibility of a polar dielectric?

To relate dielectric constant and susceptibility

$$\epsilon E = \epsilon_0 E + P = \epsilon_0 E + \chi_e E$$

$$\varepsilon = \epsilon_0 + \chi_e$$

$$\frac{\epsilon}{\epsilon_0} = 1 + \frac{\chi_e}{\epsilon_0}$$

$$\frac{\epsilon}{\epsilon_0} = K \; the \; dielectric \; constant$$

Summary

You have learnt in this module that-

- Electrostatics field E is zero in the interior of a conductor;
- Just outside the surface of a charged conductor, E is normal to the surface given by- E $=\sigma/\epsilon_0\hat{n}$ where $\hat{n}$ is the unit vector along the outward normal to the surface and σ is the surface charge density.
- Charges in a conductor can reside only at its surface.
- Potential is constant within and on the surface of a conductor.
- In a cavity within a conductor (with no charges), the electric field is zero.
- A **dielectric** (or **dielectric material**) is an electrical insulator that can be polarized by an applied electric field.
- When a dielectric is placed in an electric field, electric charges do not flow through the material as they do in an electrical conductor but only slightly shift from their average equilibrium positions causing **dielectric polarization**.
- Because of dielectric polarization, positive charges are displaced in the direction of the field and negative charges shift in the opposite direction. This creates an internal electric field that reduces the overall field within the dielectric itself. If a dielectric is composed of weakly bonded molecules, those molecules not only become polarized, but also reorient so that their symmetry axes align to the field.
- The effect of dielectric (polar or non-polar molecule) is that the total field in a medium reduces.